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Constraints on Composite Models of Quarks and Leptons

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ABSTRACT

The previously noted difficulty of obtaining Dirac magnetic moments in composite models is combined with the observation that a "light" bound fermion state with a small size must have the Dirac moment in a renormalizable theory since its anomalous moment is determined by its excitation spectrum. New constraints on composite models are given, including the decoupling of low-lying excitations and the "superconfinement" condition that creation of virtual electron-positron pairs by the superstrong gluons responsible for binding the constituents of the electron must be strictly forbidden in photon electron scattering.

The difficulty of obtaining the Dirac magnetic moment[1,2] has been suggested as a crucial test for composite models of leptons and quarks.[3] A number of papers[4-7] have subsequently argued that the Dirac moment is automatically obtained in any model which correctly describes the strong binding and small size of the leptons. This paper examines these arguments and considers four constraints which must be satisfied by any model:

1. The magnetic moment of the bound state must depend only upon the total charge of the system and be independent of how the charge is distributed between the constituents.

2. The flavor changing transition magnetic moment must vanish.

3. The excitation of multilepton states in photon-lepton scattering must be completely described by QED, with no additional contributions observable at present energies from diagrams like Fig. 1 which dominate deep inelastic electron-hadron scattering or from other effects of the superstrong interactions responsible for the binding of the constituents of the leptons.

4. The composite model for the spin 1/2 leptons must not have a low mass excitation with spin 3/2 analogous to the Δ in the quark model for the nucleon.

The difficulties imposed by the first constraint are most strikingly illustrated in the example of an electron model as a composite of a neutral fermion and a scalar boson with charge $-e$. The naive nonrelativistic model gives zero magnetic moment since the charged constituent has no angular momentum and the constituent with spin has no charge. References 4,5 suggest that the charged boson has just the right peculiar value of orbital angular momentum in a light bound state constructed from a heavy scalar boson and a heavy fermion to give the Dirac moment for the combined system. The anomalous magnetic moment and the excitation spectrum are determined by the size of the system, whereas the Dirac moment is determined by the mass or Compton wave length.

This argument is completely independent of the electric charges of the individual constituents If it holds for a neutral fermion and a charged boson, it must also hold, with the same wave function for the composite system, for a charged fermion and a neutral boson, or for a fermion with charge xe and a boson with charge $-(1 + x)e$, where x can have any arbitrary value. This extreme condition suggests that any composite model made from two different elementary fields cannot be simply described in terms of constituents, like the constituent quark model for hadrons, nor by relativistic models with the Dirac equation in external potentials. Calculations obtaining the Dirac moment for a bound fermion in an external potential[7-9] are misleading, because the potential source is assumed to be neutral and spinless. If the source is charged and the fermion is neutral the magnetic moment is zero. If the source is neutral, but is an infinitely heavy vector boson, the total angular momentum of the system is opposite to the angular momentum carried by the fermion, and the magnetic moment has the wrong sign compared to the Dirac moment for the composite system.

The constraints imposed by requiring the magnetic moment to be independent of the charge distribution between the constituents can be stated relativistically without assuming simple constituent models. Consider a composite model constructed from two basic fields, denoted by Ψ_1 and Ψ_2 with electric charges q_1 and q_2. These may be either Bose or fermi fields, but at least one fermi field is necessary to make a composite fermion. The only dynamical variables are the two fields ψ_1 and ψ_2 and there is no simple description in terms of particles carrying spin and orbital angular momenta. However, some general properties of the magnetic moment are obtained by simply assuming that the contributions of the two fields to the electromagnetic current are additive and proportional to their respective charges.

$$J^\mu = q_1 \bar{J}_1^\mu(\Psi_1) + q_2 \bar{J}_2^\mu(\Psi_2), \tag{1}$$

where $\bar{J}_1^\mu$ and $\bar{J}_2^\mu$ are "reduced" currents, with the charges factored out. The total electric charge and magnetic moment of a bound state are given by

$$Q = \langle n_1 \rangle q_1 + \langle n_2 \rangle q_2, \tag{2}$$

$$\langle \vec{\mu} \rangle = q_1 \langle \vec{\mu}_1 \rangle + q_2 \langle \vec{\mu}_2 \rangle \tag{3}$$

where n_1 and n_2 and $\vec{\mu}_1$ and $\vec{\mu}_2$ are the number and reduced magnetic moment operators for each field. In the Harari Rishon model,[3] for example, n_1 and n_2 are the numbers of V and T particles in the state and take on integral values from -3 to +3 for the quarks and leptons. The wave function can contain an arbitrary number of particle-antiparticle pairs in addition to the n_1 of type 1 and the n_2 of type 2, and may not be eigenfunctions of n_1 and n_2 if charge exchange is possible between the two fields.

The angular momentum carried by each field can be defined relativistically by observing the behavior of each field separately under rotations. The total angular momentum $\vec{J}$ is the sum of the contributions from the two fields,

$$\vec{J} = \vec{J}_1 + \vec{J}_2 \ . \tag{4}$$

For a state with a well defined angular momentum, e.g. J=1/2 for quarks and leptons, Eq. (3) can be rewritten

$$\langle\vec{\mu}\rangle = \left[q_1\langle\vec{\mu}_1\cdot\vec{J}\rangle+q_2\langle\vec{\mu}_2\cdot\vec{J}\rangle\right]\langle\vec{J}\rangle/[J(J+1)] \ . \tag{5}$$

From the argument of Refs. (4,5), the magnetic moment (5) must be the Dirac moment if the bound state has a much lower mass than the constituents, independent of the values of q_1 and q_2. The ratio of the magnetic moment (5) to the total charge Q must then be independent of q_1 and q_2. This gives following condition,

$$\langle\vec{\mu}_1\cdot\vec{J}\rangle/\langle n_1\rangle = \langle\vec{\mu}_2\cdot\vec{J}\rangle/\langle n_2\rangle. \tag{6}$$

This result (6) is a precise quantitative constraint which must be satisfied by any model which makes a light bound state out of two heavy fields with a non-electromagnetic superstrong interaction.

In all simple models, $\langle\vec{\mu}_1\cdot\vec{J}_1\rangle$ and $\langle\vec{\mu}_2\cdot\vec{J}_2\rangle$ have the same sign, i.e. the magnetic moment of a positively charged field is parallel to the direction of its angular momentum. Then

$$\frac{\langle\vec{\mu}_2\cdot\vec{J}\rangle}{\langle\vec{\mu}_1\cdot\vec{J}\rangle} = \frac{C_2\langle\vec{J}_2\cdot\vec{J}\rangle}{C_1\langle\vec{J}_1\cdot\vec{J}\rangle} = \frac{\langle n_2\rangle}{\langle n_1\rangle} > 0 , \tag{7}$$

where C_1 and C_2 are positive. Thus $\langle\vec{J}_1\cdot\vec{J}\rangle$ and $\langle\vec{J}_2\cdot\vec{J}\rangle$ have the same sign and are both less than $J(J+1)$.

$$0 < \langle\vec{J}_1\cdot\vec{J}\rangle < \langle(\vec{J}_1+\vec{J}_2)\cdot\vec{J}\rangle = J(J+1) . \tag{8}$$

A state with $J=1/2$ which satisfies this condition (8) cannot be an eigenfunction of both J_1 and J_2 and must have components both with $J_1=J_2+1/2$ and $J_1=J_2-1/2$.

The wave function defined in Ref. 2, Eq. (6) satisfies these constraints, since it was constructed to give a Dirac moment for all values of the charges of the constituents. However, as noted there, it can only be achieved with a peculiar relation between spin and statistics for the fundamental fields. More realistic models, if they exist, must have wave functions very different from those of simple constituent models; e.g. they could contain additional particle-antiparticle pairs with non-trivial angular momenta and significant contributions to the magnetic moment.

The second constraint of flavor conservation in electromagnetic transitions is trivially satisfied when all leptons are elementary. But in models where generations are different spatial excitations of the same constituents, flavor is no longer conserved, and unobserved finite electromagneitc contributions occur in processes like $\mu \to e\gamma$, $e^+e^- \to \mu^+e^-$, $e^+e^- \to s\,\bar{d} \to K^-\pi^+$, $K^0 \to 2\gamma$, $D^0 \to 2\gamma$ and the K_L-K_S mass difference. Each of these flavor-breaking transitions involves a matris element of the electromagnetic current between two fermion states of different flavors, represented in these models by two different excitations of the same constituents. Dynamical arguments involving overlap integrals of wave functions[6] have been given to suppress $\mu \to e\gamma$, based on the small size of the bound state and the low momentum of the photon. But it is not obvious that such an argument is valid for all such flavor-changing transitions. In particular there may be difficulties in processes where a lepton pair is created and there is no simple overlap integral between initial and final state wave functions, or in processes like $K^0 \to 2\gamma$ via the anomaly or the K_L-K_S mass difference which are sensitive to higher momentum components of the wave functions.

The third constraint requires complete decoupling of the low-lying excitation spectrum. The lowest excited states with the same quantum numbers as the electron have a single electron and several electron-positron pairs. Any scattering amplitude in which the electron appears as a pole must have branch points at masses of $(2n+1)m_e$ beginning with $3m_e$. The treatments of Refs. 4-7 do not consider these branch points and assume that above the electron pole the dominant contribution to photon-electron scattering in lowest order in α comes from states at very high mass Many orders of magnitude above the masses of millions of open channels allowed by all known conservation laws. These multielectron states introduce unwanted low-mass contributions into the dispersion relations and sum rules of refs. 4 and 5.

The neglect of these contributions is natural if the electron is elementary and all its interactions are described by QED. It is also required by the experimental data on photon-lepton scattering. But in a composite electron model it implies that the superstrong "gluons" which bind the constituents into a single electron are somehow forbidden to be emitted by an electron and to create electron-positron pairs. In S-matrix language this means discarding millions of known nearby singularities in the scattering amplitude and using an amplitude with an entirely different analytic structure. This also occurs in simple relativistic models based on the Dirac or Bethe-Salpeter equations.

This point is illuminated by comparison with the analogous process of photon hadron scattering in the quark-parton model described by QCD. Diagrams like those of Fig. 1 give the dominant contribution to deep inelastic photon-hadron scattering. The photon is absorbed by a quark-parton which creates additional parton-antiparton pairs by the emission and absorption of gluons and produces a multihadron final state. But the analogous diagram in photon scattering by a composite lepton leads to the unobserved process of multilepton production by pair creation of constituent partons via superstrong gluons after one parton has absorbed the photon. The superstrong gluons which bind rishons into leptons must behave very differently from the colored gluons of QCD and cannot be allowed to be emitted by partons and subsequently create parton-antiparton pairs.

The fourth constraint requires any spin 3/2 excitation to be either eliminated or pushed up to very high mass. In simple constituent models where the electron spin of 1/2 is obtained by coupling several non-trivial constituent spins to a total spin of 1/2, the spin 3/2 state arises. In the general two-field model described by Eqs. (1-7) the same problem arises even though there may not be well defined constituents. Equations (2) and (7) show that the angular momenta $\vec{J}_1$ and $\vec{J}_2$ carried by the two fields are both finite and can therefore be coupled to a total angular momentum of 3/2 as well as to 1/2. Such a spin 3/2 state destroys the sum rule arguments for a small anomalous moment. The N-Δ transition for example gives a large contribution to any sum rule for the nucleon anomalous moment.

This discussion can be summarized as requiring any composite model describing the electron to be "superrelativistic" with "superconfined" constituents.

Super-relativistic goes beyond both nonrelativistic and simple relativistic models. A non-relativistic composite model is characterized by constituent velocities $v \ll c$. Relativistic potential models using Dirac or Bethe-Salpeter equations are useful when velocities are no longer small, but when an excitation spectrum exists with energies smaller than the energy required to produce many bound state pairs. The composite model needed to describe the electron can be called superrelativistic because it must have a rich low-lying spectrum of multiparticle states and no radially excited states which could be produced with electrons in flavor changing reactions. Models where it is much easier to create many pairs than to excite the original constituents to a radial or orbital excitation cannot be described in any simple way by potential models.

Superconfinement goes beyond the ordinary confinement of QCD. Quarks in QCD are not observable as free particles, but are observable as hadrons jets produced in collisions, are emitted in pairs in hadron decays by interactions arising from QCD gluons, and give rise to forces and scattering between hadrons resulting from quark or gluon exchange. The constituents of composite leptons are confined much more than in QCD and cannot give observable effects in lepton-lepton and lepton-photon scattering outside QED. There can be no lepton jets produced by deep inelastic photon absorption on a charged constituent of the electron as in Fig. 1, and no observable electron-electron interactions resulting from superstrong gluon or constituent exchange. The superstrong interactions which bind the constituents must confine all low energy secondary effects of these superstrong interactions normally observed in QCD. Hand waving size or scale arguments are not sufficient to prove this point. QCD hadron physics has a characteristic scale of 1 GeV, but its effects in strong interactions are seen at very low energies in the scattering of thermal neutrons. And pair production effects, even when limited to short distances comparable to the lepton size, must appear in the bound state dynamics as additional contributions not included in potential or Bethe-Salpeter models.

One possible decoupling mechanism for superconfinement of superstrong gluons is the large N limit whose features were first pointed out in a simplified model[10] in 1968. With N colors and a one-gluon-exchange Yukawa potential, the effective interaction V_{eff} is proportional to Ng^2 for a color singlet state, but only to g^2 for a color uncorrelated pair. In the limit $N \to \infty$, $g^2 \to 0$, but with $V_{eff} \propto Ng^2$ fixed, the binding energy of the color singlet state remains constant, but there are no interactions between bound color-singlet states. There is a complete decoupling of the superstrong interaction. It is superstrong only inside color singlet particles and does not leak out. A model using this large N effect is currently being investigated.[11]

If such a decoupling can be achieved, two possible models are suggested: 1) a nonrelativistic potential model with very heavy constituents;[6] 2) low-lying excitations of high mass fields, like Goldstone bosons or fermions, which have zero mass in some approximation and have no simple description in terms of constituents. The Katz model[12] of strong binding of slow particles shows that light bound states of heavy particles in nonrelativistic motion are consistent with classical relativity. However, the potential V must satisfy the following conditions if the potential energy is to cancel nearly all of the constituent rest mass M and the kinetic energy T describes nonrelativistic motion:

$$\langle V \rangle \simeq M \qquad (9a)$$

$$\langle r\, dV/dr \rangle = 2\langle T \rangle = p^2/\mu \ll \mu < M \simeq \langle v \rangle \qquad (9b)$$

where the virial theorem is used to obtain Eq. (9b) and the reduced mass μ which appears in the equations of internal relative motion depends upon the Lorentz character of the potential. For vector potentials μ is of the order of the constituent mass M, as in nonrelativistic mechanics; for scalar potentials μ is of the order of the bound state mass m_b. The same Lorentz dependence of μ is obtained in simple quantum-mechanical models.[8,9] The condition (9b) requires a potential like a rounded square well with a small derivative at the bottom which is not simply related to the well depth. This excludes simple potentials like coulomb, square well, logarithmic or coulomb plus linear.

In quantum mechanics, the uncertainty principle introduces another constraint,

$$\mu^2 >> \langle p^2 \rangle \simeq 1/\langle r^2 \rangle >> m_b^2. \qquad (9c)$$

This constraint cannot be satisfied by a scalar potential model which has $\mu \simeq m_b$. The vector case is still allowed, and the constraint (9c) defines three length scales. The size of the bound state is intermediate between the scales defined by the constituent mass and the bound state mass. These conditions suggest a model like the electroweak theory with eventually observable heavy gauge bosons, analogous to W and Z but much heavier, giving rise to weak short range forces at low energies, rather than like QCD with asymptotic freedom and confinement. But how such a theory can produce an effective potential satisfying (9b) is completely unclear at present. It may even be impossible.

Decoupled superrelativistic superconfined systems naturally have the Dirac magnetic moment to a very good approximation. If all effects of the composite structure are superconfined, the anomalous moment must be very small with a mass scale determined by the excitation energy of the composite structure. But superconfinement will undoubtedly be harder to test and prove in any proposed model than confinement in QCD. Thus magnetic moment calculations may prove to be highly significant test of such models.

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FOOTNOTES AND REFERENCES

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FIGURE CAPTIONS

Fig. 1 Deep Inelastic Photon Scattering in a Parton Model.

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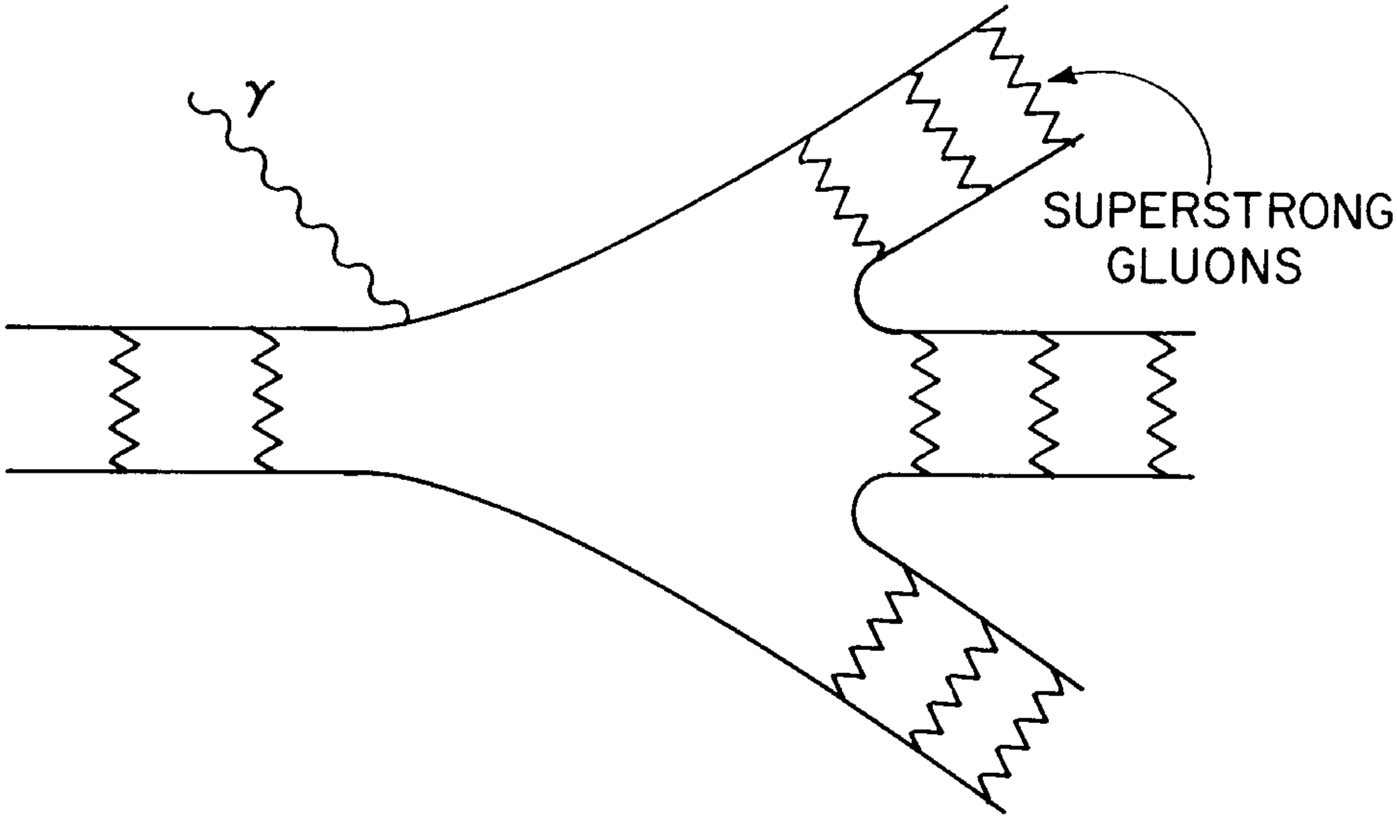

Fig. 1